6.1: Revisiting Quadratic Functions

- The graph of a quadratic function is a U-shaped curve called a **parabola**. See diagram below.
- Recall that a quadratic function must have x^2 as its highest exponent: $y = ax^2 + bx + c, a \neq 0$.
- A property of quadratic functions is **congruency**. Congruent parabolas have the same shape; this means that one parabola can be placed over the other and you would only see one parabola.
- The graphs of quadratic functions are **congruent** if the coefficients of the x^2-term in their equations are equal in magnitude (the numerical values are equal regardless of the sign).
- Example: Sort the following quadratic functions into groups of congruent parabolas.

a) $y = x^2$ b) $y = \frac{1}{4}x^2 + 7x - 4$ c) $y = -x^2 + 3x - 2$

d) $y = x^2 + 5x$ e) $y = 3x^2 + 5$ f) $y = -\frac{1}{4}x^2$

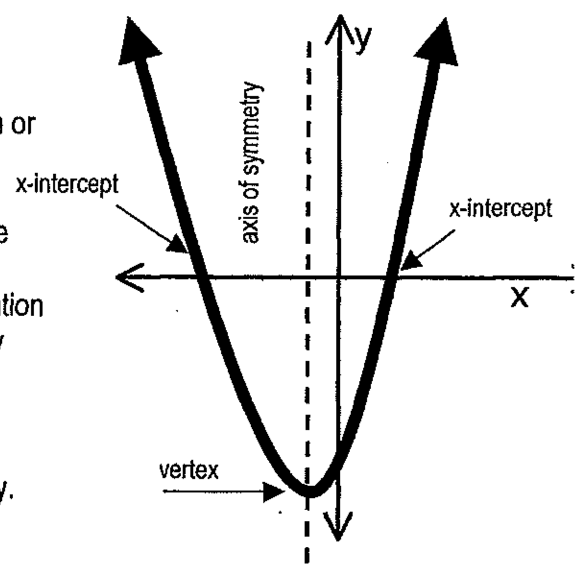
g) $y = -3x^2 + 3x - 9$ h) $y = \frac{1}{4}x^2 - 1$ i) $y = -3x^2$

Group 1: ___________ Group 2: ___________ Group 3: ___________

6.2: Sketching the Graph of $y = ax^2 + bx + c$

The original graph of a parabola is based on the equation $y = x^2$.

Graphing Parabolas



- The **vertex** is the point at which the curve changes direction. It is the point where the axis of symmetry intersects the parabola. It can be the maximum or minimum point of a parabola.
- The **axis of symmetry** is a vertical line that passes through the vertex. The graph of a quadratic function is symmetrical about this line which means it divides the parabola into two equal parts. Since it is a vertical line, the equation for the axis is the line $x = p$, where p is the point where the axis of symmetry crosses the x-axis.
- The **x-intercept(s)** of the parabola are the same as the roots of a quadratic equation. The x-intercepts are the same distance from the axis of symmetry.

Recall from Section 5.8:
- If the discriminant is positive ⟶ there are 2 roots (or x-intercepts).
- If the discriminant is zero ⟶ there is 1 root (or x-intercept).
- If the discriminant is negative ⟶ there are no real roots. This means the parabola does not cross the x-axis.

Example 1: Determine the coordinates of the vertex of the parabola with the given information.

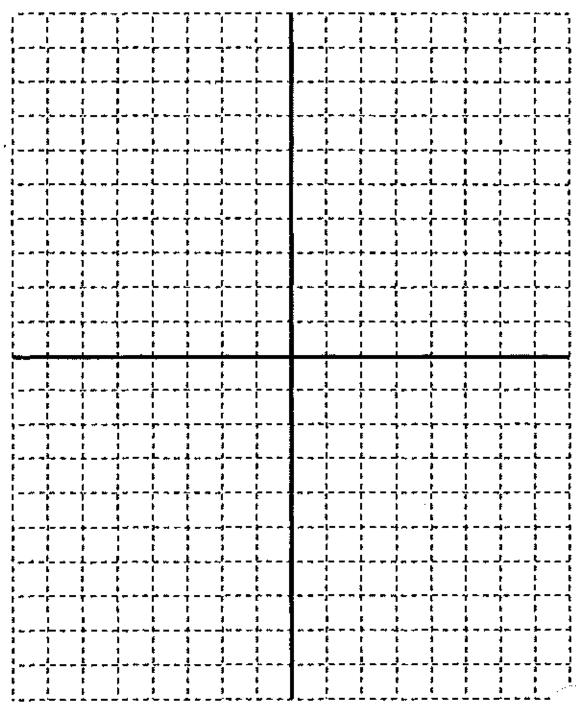
$y = -2x^2 + 4x + 6$ x-intercepts -1 and 3

Steps for Sketching the Graph of $y = ax^2 + bx + c$:

- Note: This method of sketching a parabola only works if the parabola intersects the x-axis.

① Calculate the discrimant to check if there are roots.
② Let $y = 0$, then solve the quadratic equation by factoring.
③ The roots of the equation are the x-intercepts of the parabola.
④ For the x-coordinate of the vertex: calculate the average of the x-intercepts (the midpoint of the intercepts)
⑤ For the y-coordinate of the vertex: substitute the x-coordinate from ④ into the original equation
⑥ State the vertex.
⑦ Plot the intercepts and the vertex, and then draw a smooth curve through the points. It is easier to start at the vertex, draw a curve through one x-intercept, return to the vertex and then draw a curve through the other x-intercept.
⑧ Label the parabola with the equation.

Example 2: Sketch a graph of the function $y = x^2 + 2x - 8$.



Using Technology to Investigate Transformations of Quadratics

(adapted from *Nelson Mathematics 10*)

In this lesson you will investigate how you can transform the graph of $y = x^2$ to obtain the graphs of relations in the form $y = a(x - p)^2 + q$.

Part 1:

Part 2: The Graph of $y = ax^2$ compared to $y = x^2$

Think, Do, Discuss

1. Graph the relation $y = x^2$. What is the direction of opening? __________

   What are the coordinates of the vertex? __________ What is the equation of the axis of symmetry? __________

2. On the same set of axes, graph $y = 2x^2$.

   How does the shape of the new graph compare to the previous graph of $y = x^2$? ____________________

3. Suppose that you are not using graphing technology and you have to create tables of values for the original relation and the new, transformed relation. How will the new table of values differ from the original table of values?

   ____

4. to examine the tables of values for both graphs. Was your prediction in #3 correct? __________

5. Explore what happens to the graph and the table of values when you change the coefficient, ***a***, in $y = ax^2$ from 2 to some other number. Try numbers:

   - between 0 and 1: ____________________
   - greater than 1: ____________________
   - less than 0: ____________________

6. (a) What is the same about all of the graphs? ____________________

   (b) How can you tell from the equation whether the graph will open upward or downward? __________

   (c) What values of ***a*** make the graph of $y = ax^2$ narrower than $y = x^2$? __________

   For these cases, are you vertically stretching or compressing the graph of $y = x^2$? __________
   Use the table of values to explain why this is so.

   ____

   ____

   (d) What values of ***a*** make the graph of $y = ax^2$ wider than $y = x^2$? __________

   For these cases, are you vertically stretching or compressing the graph of $y = x^2$? __________

Part 3: The Graph of $y = ax^2 + q$ compared to $y = x^2$

Think, Do, Discuss

1. clear all of the equations from part 2 except $y = x^2$. On the same set of axes as before, graph $y = x^2 - 4$.

   How does the shape of the new graph compare to the previous graph of $y = x^2$? ______________________

2. When you subtract 4 from $y = x^2$, what is the effect on the original graph? ______________________

   ____

   to examine the tables of values of both relations to see if this is true for every ordered pair. __________

3. Explore what happens to the graph and the table of values when you change the constant term q in $y = x^2 + q$ from -4 to other numbers, both positive and negative.

   What changes on the graph? ______________ What stays the same? ______________

   What effect does the constant term q have on the graph? ______________________

4. Explore the effect of varying both a and q on the graph of $y = ax^2 + q$. Fill in the table below for each graph you do.

equation		
direction of opening		
width of the parabola opening compared to $y = x^2$		
vertical stretch or compression compared to $y = x^2$		
vertex		
equation for the axis of symmetry		
number of x-intercepts		

5. Based on your explorations, how can you use the values of a and q to tell how many x-intercepts the graph of $y = ax^2 + q$ will have? ______________________

   ____

   ____

Part 4: The Graph of $y = a(x-p)^2$ compared to $y = x^2$

Think, Do, Discuss

1. , clear all of the equations from part 3 except $y = x^2$. On the same set of axes, graph $y = (x-3)^2$.

   How does the shape of the new graph compare to the previous graph of $y = x^2$? ____________________

   What is the effect on the original graph? ____________________

   examine the tables of values of both relations to see if this is true for every ordered pair. ________
   (look at the x-values when y = 0 for each graph; these are the x-intercepts)

2. Explore what happens to the graph and the table of values when you change the constant term ***p*** in $y = (x-p)^2$ from ***3*** to several other numbers, both positive and negative.

   What changes on the graph? ____________________ What stays the same? ____________________

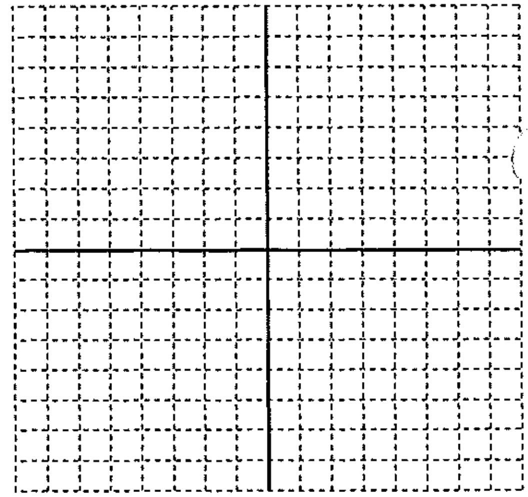
   What effect does the constant term ***p*** have on the graph? ____________________

   ____

3. Explore the effect of varying both ***a*** and ***p*** on the graph of $y = a(x-p)^2$. Fill in the table below for each graph you do.

equation		
direction of opening		
width of the parabola opening compared to $y = x^2$		
vertical stretch or compression compared to $y = x^2$		
vertex		
equation for the axis of symmetry		
number of x-intercepts		

Part 5: The Graph of $y = a(x-p)^2 + q$ compared to $y = x^2$



1. What would you have to do to the graph of $y = x^2$ to obtain the graph of $y = 2(x-5)^2 - 4$?
   List the changes below and sketch your prediction at the right.

2. On your calculator, clear all of the equations from part 4 except for $y = x^2$.
   On the same set of axes, graph the relation $y = 2(x-5)^2 - 4$.
   Was your prediction correct? ______
   If no, list the corrections.

3. What effect do the three transformations have on the original graph of $y = x^2$?

   - the effect of ***a***: ______________________________
   - the effect of ***p***: ______________________________
   - the effect of ***q*** : ______________________________

4. Explore the effect of varying both ***a***, ***p*** and ***q*** in $y = a(x-p)^2 + q$ on the graph of $y = x^2$. Fill in the table below for each graph you do.

equation		
values of a, p, and q		
direction of opening		
vertical stretch or compression compared to $y = x^2$		
vertex (p, q)		
equation for the axis of symmetry (x = p)		
number of x-intercepts		